

# **GRAPH THEORY**

## **Tutorial – 10**

1. Prove that  $\chi(G) \cdot \chi(\bar{G}) \geq n(G)$ . Use this to prove that  $\chi(G) + \chi(\bar{G}) \geq 2\sqrt{n(G)}$ .
2. Prove that  $\chi(G) + \chi(\bar{G}) \leq n(G) + 1$ .
3. Let  $G$  be an  $n$ -vertex graph and let  $c = (n+1)/\alpha(G)$ .
  - (a) Use the result in (2) and prove that  $\chi(G) \cdot \chi(\bar{G}) \leq (n+1)^2/4$ .
  - (b) Use this result to prove that  $\chi(G) \leq c(n+1)/4$ .
4. Prove that *Brook's Theorem* is equivalent to the following statement:  
“every  $k-1$ -regular  $k$ -critical graph is a complete graph or an odd cycle”.